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Mechanical reasoning by mental simulation

Mary Hegarty

Department of Psychology, University of California, Santa Barbara, Santa Barbara, CA 93106, USA

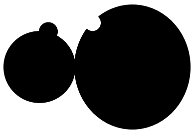
Recent studies have provided evidence for mental simulation as a strategy in mechanical reasoning. This type of reasoning can be dissociated from reasoning based on descriptive knowledge in that it depends on different abilities and memory stores, is expressed more easily in gesture than in language, exhibits analog properties, and can result in correct inferences in situations where people do not have correct descriptive knowledge. Although it is frequently accompanied by imagery, mental simulation is not a process of inspecting a holistic visual image in the 'mind's eye'. Mental simulations are constructed piecemeal, include representations of non-visible properties and can be used in conjunction with non-imagery processes, such as task decomposition and rule-based reasoning.

What would happen if you filled your bathtub to the brim with water and then stepped into the bath? If the gear on the left in Figure 1a is turning clockwise, which way will the gear on the right turn? When people answer these questions, they make mechanical or physical inferences. An inference is a cognitive process in which new information is derived from given information, and mechanics (a branch of physics) is the science of motion. A mechanical inference is therefore any mental process that allows us to derive information about how things move. Previous research in cognitive science has provided important insights into mechanical reasoning by characterizing naïve physics understanding [1,2] on which it is often based, and by developing artificial intelligence models that capture the qualitative nature of informal mechanical reasoning [3,4].

This review focuses on how people mentally represent mechanical systems and the mental processes that operate on these representations when people make mechanical inferences. This process is often referred to as running a 'mental model' of a mechanical system. There have been two different senses of mental model in the psychological literature. In one sense, a mental model is a characterization of the knowledge and cognitive processes that allow humans to understand, reason about, and predict the behavior of complex physical systems [5]. This sense of mental model does not make any strong predictions about the format of the knowledge representations involved. On this view, inferences can involve drawing analogies to familiar situations, for example, using a memory of someone diving into a swimming pool that was very full [5] to answer the bathtub question at the beginning of this article, or they can also be based on rules of mechanical reasoning, for example, the rule that interlocking gears move in opposite directions [6].

In another sense of mental model, often adopted in studies of reasoning [7] and text comprehension [8], a mental model (or situation model) is a representation that is isomorphic to the physical situation that it represents and the inference processes simulate the physical processes being reasoned about. When solving mechanical reasoning problems, many people, including famous scientists and engineers, often report the conscious

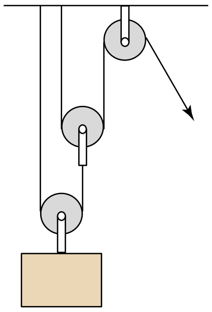
(a) Gear rotation problem



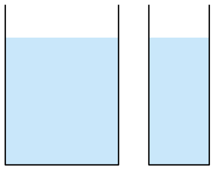
The diagram shows two interlocking gears. Will the knob on the leftmost gear mesh with the groove on the right gear if the gears are rotated inward?

(b) Pulley problem

The diagram depicts a pulley system. When the free end of the rope is pulled, will the lower pulley turn clockwise?



(c) Water pouring problem



The diagram shows two glasses of water. The glasses are the same height and filled to the same water level. If the glasses are tilted, will the water pour out of the two glasses at the same or different angles of tilt? If they are tilted at the same rate, which will pour first?

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Figure 1. Examples of the type of mechanical reasoning problems considered in this review. In each problem, the task is to infer the behavior of a mechanical system from a visual-spatial representation (a static diagram) of the mechanical system, which provides information about the shape of its components and their connectivity. Many of the items in tests of mechanical ability are of this type. **(a)** Reproduced with permission from [23]; **(b)** with permission from [28]; **(c)** with permission from [26].

Corresponding author: Mary Hegarty (hegarty@psych.ucsb.edu).

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Box 1. Spatial versus descriptive representations

A representation is something that stands for something else. To understand the nature of a representation, we must specify the *represented* world (i.e. the referent), the *representing* world (i.e. the representational medium) and the *mapping* between these two worlds [53,54]. We must also specify what aspects of the represented world are represented, because all representations involve some abstraction and thus emphasize some aspects of the information over others.

In mechanical reasoning, the represented world is essentially spatial because mechanics is the science of motion and motion is a spatial property. Mechanical reasoning depends on visible spatial information, such as the locations of objects, their shapes and connectivity. It also involves reasoning about non-visible properties, such as force, that are not visible, but are spatially distributed.

In a spatial (depictive or diagrammatic) representation, the representing world is also a spatial array. Although spatial representations can represent both spatial and non-spatial information (e.g. graphs represent number), in the mechanical domain, they represent spatial properties. This means that the spatial relations between objects in the representation correspond to the spatial relations between the objects that they represent. For example, the spatial representation in Figure I might represent five gears in a gear chain (note that this representation is abstract in the sense that it does not represent much of the visible appearance of the gears).

In a descriptive ('sentential' or 'propositional') representation the representing world is a set of facts or assertions about the represented world. These might be expressed in sentences, such as the following:

There are 5 gears in a row.
Each gear meshes with the gear(s) on either side of it.
The gear on the left is turning clockwise.

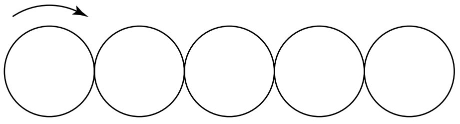

Figure I. An example of a spatial representation (see text for details).

In this case the mapping between the represented and representing worlds is arbitrary and based on conventions (e.g. the meaning of the words 'gear' and 'five').

Inferences from spatial and descriptive representations involve different cognitive processes. Inferences from spatial representations result from spatial transformations, such as mental rotation and translation. For example, the person might infer the motion of the gears in a gear chain by mentally rotating the successive gears. This corresponds to what we mean by mental simulation. By contrast, inferences from descriptive representations depend on inference rules, for example the rule 'gears that mesh with each other turn in opposite directions'. This rule might also be used to infer the motion of the successive gears.

experience of mentally simulating what will happen [7,9–11]. For example, Tesla reported that when he first designed a device, he would run it in his head for a few weeks to see which parts were most subject to wear [11]. However, these accounts are anecdotal and based on subjective reports. Recent studies are providing more objective evidence that mental simulation is sometimes used in mechanical reasoning. This research suggests that mental simulation is based on internal spatial representations of mechanical systems (see Box 1), involves analog imagery, and can be dissociated from reasoning based on descriptive representations or explicit knowledge. At the same time, new research on mental imagery [12] and new theories of mental representation [13,14] are allowing us to specify more closely the nature of mental imagery and simulation processes. It is timely to review the literature on mental representations in mechanical reasoning in this light.

Spatial representations in mechanical reasoning

Individual differences

Analysis of individual differences provides one type of evidence that mechanical reasoning can depend on spatial representations. There is a strong dissociation between spatial and verbal ability. Spatial ability has been characterized as the ability to construct, maintain and transform spatial representations accurately [15]. Solving mechanical reasoning problems, such as the pulley problem shown in Figure 1b, is highly correlated with measures of spatial ability [16,17], but not significantly correlated with verbal ability. This provides preliminary evidence that mechanical inference involves transformations of spatial representations and depends less on verbal representations.

Dual-task studies

The dual-task methodology, often used in studies of working memory, provides another type of evidence for spatial representations in mechanical reasoning. Baddeley has proposed separate buffers in working memory, specialized for maintaining visuospatial representations and verbal representations, respectively [18]. In dual-task studies we measure the amount of interference between a primary task of interest (e.g. mechanical reasoning) and different secondary tasks assumed to depend on these working memory buffers. Sims and Hegarty [19] measured the interference between mechanical reasoning and maintenance of a visuospatial working memory load (assumed to involve the visuospatial buffer) versus a verbal working memory load (assumed to tap the verbal buffer). The visuospatial working memory load interfered more with mechanical reasoning than did the verbal working memory load. Similarly, mechanical reasoning interfered more with the visuospatial than the verbal memory load. These results suggest that mechanical reasoning depends on representations in the visuospatial buffer.

Protocol studies

A third type of evidence for spatial representation is provided by protocol studies in which people are asked to 'think aloud' while solving mechanical inference problems. In these studies, individuals frequently use imitative gestures to communicate how the different components of a mechanical system move, and these gestures precede verbal descriptions of the component motions [6]. Gestures are particularly associated with communication of spatial information [20,21]. The fact that gestures precede verbal descriptions of the motions suggests that the internal representation is spatial rather than verbal.

Analog processes in mechanical reasoning

Reaction times

If mechanical reasoning problems are solved by mental simulation, there should be evidence that the inference processes are analogous to the physical processes that they simulate. Evidence for analog imagery processes was first provided by Shepard and Metzler [22] who showed that mental rotation of objects is analogous to real physical rotation, in the sense that the time to rotate an object mentally is proportional to the angle of rotation. Providing evidence for analog processes in mechanical inference, Schwartz and Black [23] found that when people inferred the rate of rotation of two interlocking gears (whether a knob on one gear would mesh with a groove on the second gear; Figure 1a), their response time was also proportional to the angle of rotation. When people were trained to use an analog imagery strategy to perform the gear task, reaction-time functions mimicked those of participants in no-training conditions, suggesting that imagery is the default strategy for performing this task. When questioned after the experiments, participants reported no knowledge of what the response-time function should be, indicating that they were not merely responding in accordance with expectations about the results of the experiment. It should be noted however, that there were also conditions under which people did not use analog imagery in this task: when they were taught an analytic strategy for performing the same task, and when they were shown a schematic diagram of the gears rather than a more realistic picture (see also [24]).

Evidence for a dissociation of analog processes from explicit knowledge

To demonstrate that people are reasoning by mental simulation, it is important to show that they do not have explicit knowledge of the physical situation about which they are reasoning. Another task studied by Schwartz provides such evidence [25,26]. In this task, people had to judge the angle at which water would pour out of two glasses, a fat glass and a thin glass (Figure 1c). People performed very poorly when they answered from explicit knowledge. However if people answered by closing their eyes and rotating an empty glass (or an imaginary glass) to indicate the answer, they were almost always correct. Moreover, there was no systematic relationship between their answers in the two experimental conditions. A similar dissociation was observed in a study of the ability of 5-year-old children to infer the trajectories of falling objects [27].

Mental simulation versus visual imagery

Mental simulation is often accompanied by the conscious experience of having a mental image [9–11]. One possible account of mental simulation is that it is a process of inspecting a mental image of the physical situation. In this account, a mental simulation is a holistic, dynamic, visual image of a physical situation. Several results in the research on mechanical inference suggest, however, that this is not an adequate account of mental simulation.

Piecemeal mental simulation

First, people mentally simulate the behavior of complex mechanical systems piecemeal rather than holistically. Take, for example, the pulley system in Figure 1b. When you pull on the rope of this pulley system, all of its parts move at once. If the mental simulation process involves inspecting a holistic mental image of the system in motion, all parts of the system should move at once in the mental image. When asked to predict how a part of the system moves, a person could merely generate the image and inspect the component in question to see how it moves. In this account, time to infer the movement of a component should be approximately the same for all mechanical components.

Contrary to this, however, Hegarty found that people take more time to infer the motion of the lower pulley in Figure 1b, than the middle pulley and more time to infer the movement of the middle pulley than the upper pulley [28]. It was reasoned that people infer the motion of the components piecemeal, beginning by imagining the rope being pulled, and working through the causal chain of events in the motion of the system. Consistent with this account, when asked to infer the motion of a particular component (say the middle pulley), eye fixations indicated that people looked at that component, and components earlier in the causal chain of events (i.e. the upper rope and pulley) but not components later in that chain of events. This account is consistent with artificial intelligence models in proposing that mechanical reasoning involves sequentially propagating the effects of local interactions between components [3,29,30]. However, it differs from these accounts in proposing that the inference process at each ‘link’ in the causal chain might involve analog imagery.

Non-visual representations in mental simulation

Second, mental simulation is not based purely on visual information, but also incorporates information about non-visible entities and properties, such as force and density. For example, Schwartz [25] compared performance of the water-pouring problem depicted in Figure 1b when people were sitting holding an empty glass upright with when they were lying down, holding the glass sideways (that is, in the same relation to their bodies but in a different relation to gravity). In both cases, they were to imagine that the glass was upright, that water reached a particular level in the glass, and they were to turn the glass until the imagined water would start to pour out. Subjects were able to make the correct inference when sitting upright, but not able to do so when lying down, indicating that they could not ignore the effects of gravity. In another experiment, participants were told to imagine that the glasses contained molasses rather than water. In this case they tilted the glasses further, reporting that they were taking into account the rate at which the liquid would respond to the tilt (i.e. its viscosity).

Action and mental simulation

Third, mechanical inference is often accompanied by actions that simulate the motion of objects [6] and such actions can facilitate inference [26], suggesting that mental simulation might involve motor representations as well as visual representations [31–34]. In fact, the results of an action (real or simulated) are not always available to visual awareness. Schwartz and Black asked participants to close their eyes and tilt a glass until an imagined amount of water would begin to pour out [26]. After indicating their answer with their eyes closed, they were allowed to open them and adjust the tilt of the glass. If the mental simulation process was accompanied by an accurate visual image of how far the glass was tilted, then participants should not adjust their tilt. However, almost all of their participants did make an adjustment.

Other strategies in mechanical inference

Finally, although there is strong evidence for mental simulation as a strategy in mechanical inference, it is just one of the possible strategies used. Many mechanical inferences are made on the basis of rules of mechanical reasoning that can be easily verbalized (for examples, see [2,35]) or analogies to familiar situations [5]. Therefore mechanical reasoning is best thought of as a hybrid reasoning process that uses all of these inference processes [36]. Furthermore, mental simulation is often used in conjunction with other strategies, such as task decomposition, as in Hegarty's pulley experiments [28]. Finally, mental simulation can lead to the discovery and formulation of rules of mechanical reasoning. When Schwartz and Black [6] asked people to solve gear problems such as the one discussed in Box 1, they initially mentally simulated the motion of the individual gears, but on the basis of those simulations discovered the simple rule that any two interlocking gears move in opposite directions and switched to a rule-based strategy. Schwartz and Black proposed that people use mental simulation in novel situations in which they do not have an available rule or when their rules are inadequate (e.g. are too narrow for the situation at hand).

Knowledge representation in mental simulation of mechanical systems

Visual vs. spatial representations

If mental simulations are not holistic mental images, how might we characterize the working memory representations on which they are based? Recent studies have made a distinction between spatial and visual imagery [12,37] corresponding to neural activity in the dorsal and ventral visual systems, respectively [38]. Visual imagery represents the visual appearance of an object, such as its shape, color or brightness. Spatial imagery represents of the spatial relationships between parts of an object, the location of objects in space, or their movement, and is not limited to the visual modality (e.g. one could have an auditory or haptic spatial image). Use of spatial imagery that encodes only essential spatial relations and omits visual details is associated with more success in verbal reasoning [39] and problem solving in physics [40], mathematics [41], and chess [42]. It is also associated with high spatial visualization ability [40,41]. It is likely that mental simulations in mechanical reasoning also depend on transformations of spatial images.

Implicit vs. explicit knowledge

We have seen that mental simulations cannot be explained in terms of explicit knowledge of the physical situation, but can they be explained in terms of implicit knowledge? It might be argued that if a person infers the correct direction of motion of a gear or pulley during mental simulation, the person has, at some level, tacit or implicit knowledge of the correct answer [43,44]. This might be generic knowledge of physical constraints; for example, that solid objects that come in contact exert forces on each other, or that liquids conform to the shape of their containers. Very young babies demonstrate implicit knowledge of such physical constraints [45]. It is plausible that people fall back on this very early (perhaps innate) knowledge of the physical world in situations in which they must make mechanical inferences but do not have relevant explicit knowledge to draw on. However, what is important to the mental simulation account is that this tacit, implicit or 'deep' knowledge [6,7,46] is accessed or 'reveals itself' only during mental simulation.

Representational format

A more controversial question concerns the format of the knowledge representations underlying mental simulation [37,43]. Clearly, at some level, all representations are encoded as neural signals, and yet the conscious experience of mental simulation is very different from that of applying inference rules [7,23]. At a minimum, the representations underlying mental simulation must be more similar to the experience of perceiving a mechanical system than are descriptive representations (Box 1). One possibility is that they are perceptual, that is, they involve some of the same neural activity involved in perceiving spatial properties [13,14,46]. For example, mechanical inference might involve partial activation of high-level spatial representations. By contrast, the representations involved in rule-based reasoning might be more abstract.

Dissociations between abstract and more perceptual representations have also been found in studies of so called 'naïve physics' [1,2]. For example, in situations where people cannot accurately predict motion trajectories (e.g. of a ball ejected from a curved tube), they can sometimes detect whether an animation shows the correct trajectory, although there are also limitations on this detection [47–49]. Perceptual representations might also be involved in 'representational momentum' – the perceptual phenomenon that when people view a moving object, they tend to remember its final position as displaced forward in space [50,51]. These effects can also be dissociated from explicit knowledge of motion learned in physics classes [52].

Conclusion

In summary, recent studies have provided evidence for mental simulation as a strategy in mechanical reasoning. This type of reasoning can be dissociated from reasoning based on descriptive knowledge in that it depends on different abilities and memory stores, is expressed more easily in gesture than in language, exhibits analog properties, and can result in correct inferences in situations where people do not have correct descriptive

Box 2. Questions for future research

- Under what conditions do people use simulation rather than rule-based reasoning or analogy in mechanical reasoning? What factors, including knowledge [6] and the nature of the visual display [19,20] affect strategy choice?
- What is the nature of the internal representations underlying mental simulation? Are they represented in the visual modality? How much visual detail do they include? Do they also include information from other modalities, that is, are they multimodal?
- How can we best characterize individual differences in mental simulation ability? Do they reflect differences in the capacity of spatial working memory, differences in executive processes, such as task decomposition, which compensate for limitations in this capacity, or differences in knowledge of the mechanical situations to be simulated.
- How complex a mechanical system can be simulated by spatial imagery? Does working memory limit the mental simulation of complex systems? Are some types of motion more difficult to simulate than others (e.g. rotation vs. translation)?
- How does mental simulation change with expertise in a domain? Does expertise allow the mental simulation of more complex mechanical systems? With expertise, is mental simulation replaced by other forms of reasoning?
- What are other possible situations in which people use mental simulation in thinking? To what extent can the research on mental simulation in mechanical reasoning be generalized to these situations?

knowledge. Although it is frequently accompanied by imagery, mental simulation is not a process of inspecting a holistic visual image in the 'mind's eye'. Rather, mental simulations are constructed piecemeal, include representations of non-visible properties and can be used in conjunction with non-imagery processes, such as task decomposition and rule-based reasoning. There is much to discover about the nature of the mental representations underlying mental simulations, when it is used in mechanical reasoning, and its limitations as an inference process (see Box 2). The research to date suggests that mechanical reasoning provides an interesting domain in which to study the functions of spatial representations, and how people flexibly use these and other types of mental representations in thinking.

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