Probability Illusions

Perseverance of belief

- Suicide notes study (Ross, Lepper, & Hubbard, 1975)
  - Judge from these letters whether the person is likely to commit suicide
  - Two (false) feedback conditions
    - You were right on 24/25   Great Detector
    - You were right on 10/25   Terrible Detector
  - Participants then told—feedback bogus!
  - Asked to estimate:
    - Number of letters you were really right on
    - Number of letters you would be right on in the future

Perseverance of belief

■ "Success" feedback
■ "Failure" feedback

current   future

estimate of # correct

Discovering Probability Illusions

- Our method of research in those early Jerusalem days was pure fun. We (Danny and Amos) would meet every afternoon for several hours which we spent inventing interesting pairs of gambles and observing our own intuitive preferences. If we agreed on the same choice, we provisionally assumed it was characteristic of human kind and went on to investigate its theoretical implications, leaving serious verification for later . . . . In a few giddy months we raced through more than twenty diverse theoretical formulations. (Kahneman and Tversky, *Choice, Values, and Fames*, 2000, p. x)

Framing Effect: Context
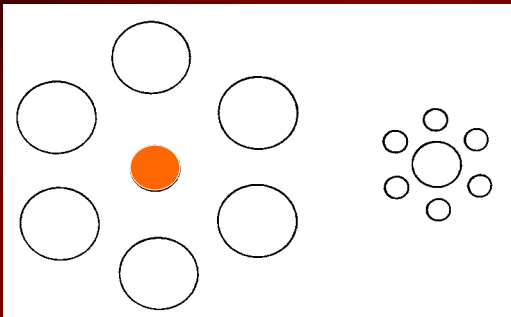
Classical Utility Theory

- Maximize expected value
- Use money as a good estimate of value
- Expected value of a choice is simply the payoff times the likelihood of getting the payoff

St. Petersburg Paradox

Peter tosses a coin and continues to do so until it should land "heads" when it comes to the ground. He agrees to give Paul one ducat if he gets heads on the very first throw, two ducats if he gets it on the second, four if on the third , eight if on the fourth, and so on, so that with each additional throw the number of ducats he must pay is doubled.
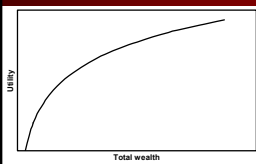
(Bernoulli, 1738)

Expected value vs. Subjective Utility

- The expected value of this gamble is infinite :
  - *EV* = ½ x 1 + ¼ x 2 + 1/8 x 4 …
    = ½ + ½ + ½ + ½ + ½ + ½ … = ∞

- However,
  - *Although the standard calculation shows that the value of Paul's is infinitely great, it has…to be admitted that any fairly reasonable man would sell his chance, with great pleasure, for twenty ducats."*
  - That is, although the expected value of the gamble is infinite the subjective utility is low, as indexed by people's willingness to sell their chance to play for a small sum.

The utility of wealth

Marginal declining utility

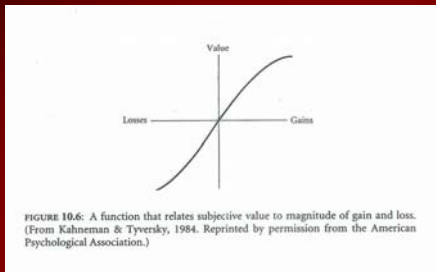*Utility* (vertical axis) / *Total wealth* (horizontal axis)

- To explain this, Bernoulli suggested that EV ≠ EU
  - Instead the utility of wealth is proportional to its logarithm.
  - Each additional unit of wealth is worth less than the previous one.
  - If I have $1000,000 I care less about an additional $500, than if I only have $1000.
- So the extra utility of the high winnings in the St. Petersburg Paradox is no longer high enough to compensate for the very low probabilities.

Expected Utility Theory

- John von Neumann and Oskar Morgenstern (1947)
  - Identify axioms of rational decision making
    - Ordering of alternatives: preference ranking
    - Dominance: don't choose a strategy that yields a better outcome for your competitor
    - Cancellation: It two alternatives share possible outcomes in common, they should make no difference
    - Transitivity: If A preferred to B, B to C, than A should be preferred to C
    - Continuity: For any set of outcomes, under some odds one should prefer a gamble between the best and worse to an intermediate outcome
   - Invariance: Choice should not be affected to manner of presenting the alternatives

Subjective Utility Curve

FIGURE 10.6: A function that relates subjective value to magnitude of gain and loss. (From Kahneman & Tyversky, 1984. Reprinted by permission from the American Psychological Association.)

Losses vs. Gains

- First, we are offered a bonus of $300. Then, we are asked to choose between the two following possibilities:
  A. To receive $100 for sure; or
  B. To toss a coin. If we win the toss, we will get $200; if we lose, we receive nothing at all.
- First, we are offered a bonus of $500. Then, we are asked to choose between the two following possibilities:
  C. We are guaranteed to lose $100.
  D. We toss a coin, and if we lose, we have to pay $200, but if we win, we don't have to pay anything.

Different values for Losses and Gains
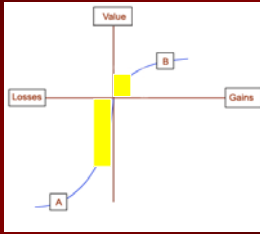
Ratio vs Absolute Difference

- Imagine that you are about to purchase a jacket for $125, and a calculator for $15.
  - You are told that the same calculator you wish to buy is on sale for $10 at the other branch of the store, located 15 minutes' drive away. Would you make the trip to the other store? **Yes**
  - You are told that the same jacket you wish to buy is on sale for $120 at the other branch of the store, located 15 minutes' drive away. Would you make the trip to the other store? **No**

Certainty Effect

- You find yourself forced to play Russian Roulette
- How much would you pay to reduce the number of bullets from 1 to 0?
- How much would you pay to reduce the number of bullets from 5 to 4?

- Richard Zechhauser found people would pay more in the first case than the second, although the benefits secured are equivalent.

Allais Paradox

- Which do you prefer:
  - A. 25% chance of winning $30 — **35%**
  - B. 20% chance of winning $45 — **65%**
- Which do you prefer:
  - C. 100% chance of winning $30 — **80%**
  - D. 80% chance of winning $45? — **20%**

- Let utility of $0=0, utility of $45=1 and 0 < utility of $30 < 1
- Choosing B over A entails that the utility .25 x the utility of $30 is less than .20 x the utility of $45, or utility of $30 is < .8
- Choosing C over D entails that the utility of 30 is greater than .8 x utility of 45, or utility of $30 > .8


Allais - 2

- Which do you prefer?
  - ★ a. $1000 $p = 1.0$
  - b. $1000 $p = 0.94$
    $5000 $p = 0.05$
    $0 $p = 0.01$

- Which do you prefer
  - c. $1000 $p = 0.06$
    $0 $p = 0.94$
  - ★ d. $5000 $p = 0.05$
    $0 $p = 0.95$

	1%	5%	94%
1a	$1000	$1000	$1000
1b	$0	$5000	$1000
2a	$1000	$1000	$0
2b	$0	$5000	$0

Segregating Decisions

- "we accept [the problem] in the terms in which it is formulated and do not seek an alternative form"
  - "we . . . Always seek to solve a problem *as presented*"

Framing in Scientific Reasoning

- Radiation problem:
  - Problem
    - Patient has an abdominal tumor (in center of body)
    - Radiation will kill tumor
    - But rays strong enough to kill tumor will also destroy healthy tissue that rays pass through on way to tumor
  - Solution?
    - Don't be upset if you cannot figure this out—only 20% solve it (Gick and Holyoak, 1980)
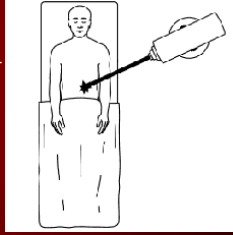
Try Another Frame

- Fortress Problem:
  - Army general wants to attack a fortress in center of forest.
  - Many roads lead to fortress like spokes of a wagon wheel.
  - All the roads have land mines that are triggered by heavy traffic, but not by light, ordinary traffic.
  - If entire army travels down a single road, its weight will cause the land mines will go off.
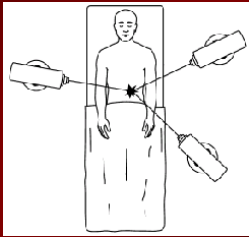
- Solution?

Solution

The general sends small groups of soldiers down each of the roads, co-ordinating their movements so that they all arrive at the fortress at the same time.

Try the New Frame on the Old Problem

Structural Similarities

Fortress Problem		Radiation Problem
Fortress	←→	Tumor
Mined Roads	←→	Surrounding tissue
Attacking troops	←→	Rays varying in intensity
Small groups of troops Converging on fortress	←→	Weak rays focused on tumor

Gick and Holyoak's Results
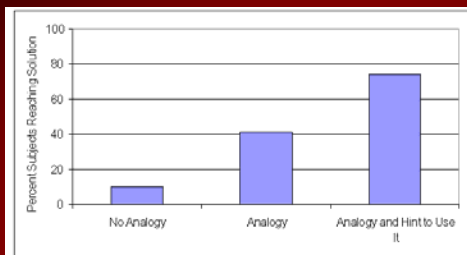
Conjunction Effect

Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Please rank the following statements by their probability, using 1 for the most probable and 8 for the least probable.

(a) Linda is a teacher in elementary school.
(b) Linda works in a bookstore and takes Yoga classes.
(c) Linda is active in the feminist movement.
(d) Linda is a psychiatric social worker.
(e) Linda is a member of the League of Women Voters.
(f) Linda is a bank teller.
(g) Linda is an insurance sales person.
(h) Linda is a bank teller and is active in the feminist movement.
Naïve subjects: (h) > (f) 89%   Sophisticated subjects: 85%

Reducing Conjunction Error

Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.
There are 100 people who fit the description above. How many of them are:

…
(f) bank tellers?
(h) bank tellers and active in the feminist movement?
...

$$(h) > (f) = 13\%$$

Base Rates

- Jim is short, slim, and likes to read poetry. Is Jim more likely to be a classics professor at an Ivy League university or a truck driver?

- How many Ivy League universities are there? 8
- How many Classics Professors at each? ≈ 4
- How many of those Classics professors are short and slim? ≈ 50%
- 4. How many of those short, slim, Classics professors like to read poetry? ≈ 50%
- How many Ivy League Classics professors fit the description? ≈ 8

Base Rates

- How many truck drivers are there? $\approx$ 400,000
- How many are short and slim? 1/8
- How many of those short, slim truck drivers like to read poetry? 1/10
- How many truck drivers fit the description? $\approx$ 5000

- What are the odds that the person in the description is a truck driver? $\approx$ 625 to 1

- Even if our estimates are off, they are not likely to be in the wrong direction

Base-Rate Neglect Examples

- Why are more graduate students first-born than second-born children?
- Why do more hotel fires start on the first ten floors than the second ten floors?
- In baseball, why are more runners thrown out by pitchers on first-base than second-base?

Framing Effect

- If doctors are told there is a mortality rate of 7% within 5 years for a certain operation, they will hesitate to recommend it to their patients.
- BUT, if they are told it has a survival rate after 5 years of 93%, they are more inclined to recommend it to their patients.

Framing Effect

- Do you think the United States should allow public speeches against democracy?
- Do you think the United States should forbid public speeches against democracy?

	Allow	Don't Forbid
1940	25	46
1974	56	72
1976	55	80

Framing Effect

- Choose either
  - A. A sure gain of $240 ⭐
  - B. A 25% chance to gain $1000 or a 75% chance to gain nothing
- Choose either
  - C. A sure lose of $750
  - D. A 75% chance to loose $1000 and a 25% chance to loose nothing ⭐

Framing Effect

A	$240	$240
D	-$1000	$0
A&D	-$760	$240

| | 75% | 25% |

B	$0	$1000
C	-$750	-$750
B&C	-$750	$250

Framing Effects

- Macro-economic policies often involve trade-offs. For example, policies intended to raise employment often have adverse effects on inflation.

Policy	Work Force Unemployed (%)	Rate of Inflation (%)
Program J	10	12
Program K	5	17

- Imagine you were faced with the decision of adopting program J or program K. Which would you choose?

J: 36%; K: 64%

Framing Effects

- Now assume that you have to choose between the following policy programs, J or K. Which would you choose?

Problem 10 (N = 133)

Policy	Work Force Employed (%)	Rate of Inflation (%)
Program J	90	12
Program K	95	17

- J: 54%; K: 46%

Loss Aversion

- Kahneman, Knetsch & Thaler (1990) divided class into two groups:
  - *Sellers* are given decorated mug to keep and asked how much they are willing to sell it for. $7.12
  - *Choosers* are asked how much money they would find as attractive as the mug $3.12
- afterwards a market price will be set.
- Same situation for both but frames differ:
  - Sellers lose their mug, chooser gain a mug.

Taxi Cab Problem

- In a city in which two cab companies, Blue and Green, operate, a taxicab was involved in a hit and run accident one night
  - 85% of the cabs in the city are Green, 15% Blue
  - A witness identified the cab as Blue
  - The Court tested the ability of the witness to identify cab colors under appropriate visibility conditions
  - He/she made the correct identification 80% of the time
  - What is the probability that the cab involved was Blue?
- Confident enough to convict?

Taxi Cab Problem - 2

	Said Blue	Said Green	Totals
Blue	12	3	15
Green	17	68	85
Totals	29	71	100

- Of the times he/she said it was Blue, it was blue 12/29 or 41%
- Is <50% accuracy good enough to convict?

Savage: Expected Utility Theory

A businessman contemplates buying a certain piece of property. He considers the outcome of the next presidential election relevant to the attractiveness of the purchase. So, to clarify the matter for himself, he asks whether he would buy if he knew that the Republican candidate were going to win and decides that he would do so. Similarly he considers whether he would buy if he knew that the Democratic candidate were going to win. and again finds that he would do so. Seeing that he would buy in either event he decides that he should buy, even though he does not know which event obtains. It is all too seldom that a decision can be arrived at on the basis of the principle used by this businessman but except possibly for the assumption of simple ordering. I know of no other extralogical principle governing decision that finds such ready acceptance (Savage 1954, p 21)

Disjunction Effect

Imagine that you have just taken a tough qualifying examination. It is the end of the fall quarter, you feel tired and rundown, and you are not sure that you passed the exam. In case you failed you have to take the exam again in a couple of months-after the Christmas holidays. You now have an opportunity to buy a very attractive 5-day Christmas vacation package to Hawaii at an exceptionally low price. The special offer expires tomorrow, while the exam grade will not be available until the following day. Would you

32% – **x** buy the vacation package **32%**
7% – y not buy the vacation package **7%**
61% – **z** pay a **$5** nonrefundable fee in order to retain the rights to buy the vacation package at the same exceptional price the day after tomorrow-after you find out whether or not you passed the exam **61%**

Disjunction Effect

Imagine that you have just played a game of chance that gave you a 50% chance to win $200 and a 50% chance to lose $100. The coin was tossed and you have [won $200/lost $100/still uncertain]. You are now offered a second identical gamble
– 50% chance to win $200 and
– 50% chance to lose $100

Would you	Won	Lost	Uncertain
x Accept the second gamble	69%	59%	36%
y Reject the second gamble	31%	41%	64%

Newcomb's Paradox

- Subject plays a game with a computer, and is told that the consumer can predict from his behavior what he will do. Two boxes appear on computer screen. Subject is told that computer knows pretty well whether subject will choose box A or both box A and B. Computer puts $1000 in box A, and $1000,000 in box B if it thinks subject will choose box B alone, otherwise puts nothing in box B.
- (named after William Newcomb of Livermore Labs by Robert Nozick, 1969, reformulated by Shafir & Tversky)