Valid Arguments

Brief Review

- **Statements** are sentences that have a **truth value**—are either true or false
- Compound statements
  - Tautologies: statements that are always true
  - Contradictions: statements that are always false
  - Contingent: statements that could be true or could be false depending on the truth value of their components
- **Arguments** are sets of statements, some of which serve as **premises** for others, which are **conclusions**

Evaluating Arguments

- Two criteria for evaluating arguments
  - Is the connection between the premises and the conclusion such that the premise would support the conclusion if they were true?
    - **Valid arguments** are arguments in which, if the premises are true, the conclusion ***must*** also be true
  - Are the premises true?
- Sound arguments are ***valid*** arguments with ***true*** premises

Valid Arguments

- A valid argument is defined by the condition: if the premises are true, the conclusion ***must*** also be true
  - Or, equivalently: A valid argument ***cannot*** have true premises and a false conclusion
- Note the words ***must*** and ***cannot***
  - These are modal terms: they tell us would be the case if the stated conditions are true
  - These conditions (premises) might not be true
    - And so the definitions say nothing about what happens when they are not satisfied

Examples: Validity and Soundness

- Consider the argument
  - You are an Independent, therefore you cannot be President of the US
    - Validity: Can the premise be true and the conclusion false?
      - Yes, so this argument is not valid
      - Consequently, it is not sound
- Consider the argument
  - The President is a human being, therefore he is a mammal
    - Validity: Can the premise be true and the conclusion false
      - No, so this argument is valid
      - Is the premise also true? Yes. So the argument is sound

Clicker Question

An argument with all true premises and a true conclusion is

A. Sound
B. Valid
C. Valid but not sound
D. Cannot tell

Clicker Question

An argument with a false conclusion is

A. Not sound
B. Not valid
C. Valid but not sound
D. Cannot tell

Clicker Question

Can a valid argument have a false conclusion?

A. No, all valid arguments have true conclusions
B. Yes, any valid argument may have a false conclusion
C. Yes, but only if it has one or more false premises
D. No, since it would not then be sound

Clicker Question

The conclusions of valid arguments with false premises are always false

A. True
B. False

Conditional Statements

- Conditional statements consist of two component statements linked by the logical connective IF, THEN
  - If the dog barks, (then) the dog will bite.
- *If* and *then* are not argument indicator words—they are not marking premises and conclusions of an argument
  - *If it rains today there will be no picnic* is not an argument!
    - It simply asserts a conditional relationship between two statements
  - Compare: *It is raining today, therefore there will be no picnic.*
    - This is an argument whose conclusion is that there will not be a picnic.

Conditional Statements - 2

- IF (antecedent), THEN (consequent) is a *truth functional* connective: the truth of a compound (whole) statement depends only on the truth values of the component statements

  ***If A, then B* is false**
  when the antecedent is true and the consequent is false.
  Otherwise, it is true.

- If you trespass, then you will be arrested
  - is false if you trespass and are not arrested
  - is true if you trespass and are arrested
  - is true if you do not trespass and are not arrested
  - is true if you do not trespass and are arrested

  The last case may seem surprising, but of course there are other reasons you might be arrested

Clicker Question

The statement "If the door is open, the alarm sounds" is false if

A. The door is open and the alarm sounds
B. The door is open and the alarm does not sound
C. The door is not open and the alarm sounds
D. The door is not open and the alarm doesn not sound

Reversing Antecedent and Consequent

- *IF A, THEN B* is **NOT** equivalent to *IF B, THEN A*
  IF A, THEN B is false when A is true and B is false
  IF B, THEN A is false when B is true and A is false

  Contrast
  If the economy has improved, we will go to war
  If we go to war, then the economy has improved

IF A, THEN B **is equivalent to** *IF not B, THEN not A.*

If you trespass, then you will be arrested
is equivalent to
If you are not arrested, then you did not trespass

Clicker Question

"If I miss this question, I need to study" is equivalent to

A. If I don't miss this question, I don't need to study.
B. If I don't need to study, I did not miss this question.
C. If I need to study, I missed this question.
D. I missed this question, therefore, I need to study.

ONLY IF

- ***IF, THEN*** versus ***ONLY IF***

Compare:
If you trespass, then you will be arrested
False if you trespass and are not arrested
Only if you trespass will you be arrested
False if you don't trespass and are arrested

B ONLY IF A is equivalent to If B, then A

If you were arrested, then you trespassed

THERE IS NO IF IN *ONLY IF*

ONLY IF - 2

- How to say "IF you are an officer, THEN you can eat in this room" with ONLY IF?
  - ONLY IF you can eat in this room are you an officer
    - Both are false if you are an officer but cannot eat in this room
- What does "ONLY IF you are an officer can you eat in this room" say?
  - IF you can eat in this room, THEN you are an officer
    - Both are false if: you can eat in this room but are not an officer

UNLESS

UNLESS can also be used to assert conditional relations
Rule: UNLESS = IF NOT

Unless you complete the assignment, you will not get promoted
says the same thing as
If you do not complete the assignment, you will not get promoted
or
If you get promoted, then you completed the assignment.

Clicker Question

Which statement is not equivalent to the others?

A. If there is a storm, the dogs will bark
B. Only if the dogs bark is there a storm
C. Only if there is a storm will the dogs bark
D. Unless the dogs bark, there is no storm

Clicker Question

Which statement is not equivalent to the others?

A. Unless there is a test, there is no need to attend class
B. If there is a test, then there is a need to attend class
C. If there is a need to attend class, then there is a test
D. Only if there is a test is there a need to attend class

Sufficient Conditions

When a conditional statement uses general terms (e.g., *dog*, *mammal*) it expresses relations between categories of things that satisfy those terms

If something is a dog, then it is a mammal

Presents a relation between ***being a dog*** and ***being a mammal***

It asserts that meeting the first condition (being a dog) *suffices* for meeting the second condition (being a mammal)

If _____, then_____

suffices for

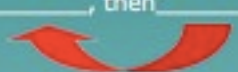
Necessary Conditions

Since a true conditional statement cannot have a true antecedent and a false consequent, the consequent of a conditional expresses something that is *necessary* if the antecedent is true

If something is a dog, then it is a mammal

Asserts that meeting the second condition (being a mammal) is necessary for meeting the first condition (being a dog)

If _____, then_____



is necessary for

Necessary and Sufficient Conditions

- Passing statistics is a necessary condition for a B.S. in psychology.
  - If a person has a B.S. in psychology, that person has passed statistics.
- Voting is sufficient for being a U.S. citizen.
  - If someone votes, then that person is a U.S. citizen.
- Believing in God is necessary to be a Boy Scout.
  - If someone is a Boy Scout, then that person believes in God.
- Not taking the exam is sufficient for failing this course.
  - If you do not take the exam, then you fail this course.

If versus *Only if* again

What follows the *If* of a conditional is a sufficient condition

What follows *only if* is a necessary condition

You can vote only if you are at least 18 years old

Being 18 is a necessary condition for voting

If you are able to vote, then you are at least 18 years old

Being able to vote is sufficient (evidence) that you are at least 18 years old

Clicker Question

Assume:

Sales are increasing = T ***Our sales force is less effective*** = F
We need to build a new plant = F ***We have excess production capacity*** = T

What is the truth value of the following statement?

Whenever sales are increasing, we need to build a new plant

A. True
B. False

Clicker Question

Assume:

Sales are increasing = T ***Our sales force is less effective = F***
We need to build a new plant = F ***We have excess production capacity = T***

What is the truth value of the following statement?

Only if sales are increasing do we need to build a new plant

A. True
B. False

Clicker Question

Assume:

Sales are increasing = T ***Our sales force is less effective = F***
We need to build a new plant = F ***We have excess production capacity = T***

What is the truth value of the following statement?

Unless we have excess production capacity, we need to build a new plant

A. True
B. False

Using conditionals in inference

There are two ways to use a conditional statement in a valid inference, one obvious, one less so:

The obvious way:
From *IF A, THEN B*, affirm A
From this it follows that B

Why?
If *B* weren't true, and A is true
If A, then B would be rendered false

So, the following form is VALID:
If A, then B
A
∴B

Modus ponens

Using conditionals in inference - 2

The second, less obvious way:

From *IF A, THEN B*, what happens if B is denied?

If *B* is false and *A* is true, then what is the truth value of IF A, THEN B?

It is false. Thus A cannot be true when the whole conditional is true. Accordingly:

If A, then B
Not B
∴Not A

is VALID *Modus tollens*

Uses of conditional arguments in scientific reasoning

Modus ponens is most commonly invoked to make predictions from a hypothesis

If malaria is transmitted by mosquitoes and we eliminate the mosquitoes, malaria will decline
Malaria is transmitted by mosquitoes and we are eliminating the mosquitoes
∴Malaria will decline

Modus tollens is most commonly invoked to confirm or falsify a hypothesis based on the truth of falsity of a prediction

Invalid conditional arguments

Not all arguments that start with conditional statements are valid

What can you conclude about B (validly) from:

If A, then B
Not A
?

Denying the Antecedent
INVALID

Remember, to be valid, it must be that *if the premises were true, the conclusion would also have to be true*

What conclusion about B has to be true in this case?
Both *B* and *not B* are compatible with the premises
There is no valid argument here!

Invalid conditional arguments - 2

What about if we start with:

If A, then B
B
?

Affirming the consequent
INVALID

What conclusion about A has to be true in this case?
Both *A* and *Not A* are compatible with these premises
There is no valid argument here either!

Clicker Question

- What form is this argument?
  - I know I passed since I took the test, and if I took the test, I passed.

A. Modus ponens
B. Affirming the consequent
C. Modus tollens
D. Denying the antecedent

Clicker Question

- What form is this argument?
  - Whenever the computer is broken, I have to calculate the result by hand. Today I had to calculate the result by hand. Thus, the computer must have been broken.

A. Modus ponens
B. Affirming the consequent
C. Modus tollens
D. Denying the antecedent

Clicker Question

- What form is this argument?
  - Only if the dog is white is the ball blue. Indeed, the dog is white. So, the ball is blue.

A. Modus ponens
B. Affirming the consequent
C. Modus tollens
D. Denying the antecedent

Reasoning with *And*, *Or* and *Not*

A very commonly used valid argument form is the following:

Either A or B
Not A [or Not B]
∴ B [or A]

Alternative Syllogism
Valid

Common reasoning strategy:

- start with an *exhaustive* set of alternatives
- eliminate all but one
- conclude that the remaining one is true

Reasoning with *And*, *Or* and *Not* - 2

An important but somewhat confusing type of inference involves negations operating on disjuncts (or) or conjuncts (and)

Consider the statement:
You cannot enlist in both the Army and the Navy

This is not the same as
You cannot enlist in either the Army or the Navy

If you want to make the statement using or you must divide the negation:
Either you do not enlist in the Army or you do not enlist in the Navy

Reasoning with *And*, *Or* and *Not* - 3

Likewise, consider the statement

> Neither San Diego nor Los Angeles will win the World Series this year

Which is equivalent to

> It is not the case that either San Diego or Los Angeles will win the World Series this year

It is not enough simply to move the *not* to be with the two parts:

> Either San Diego will not win the World Series this year or Los Angeles will not win

To maintain the meaning, you must switch to an *and*

> San Diego will not win the World Series this year and Los Angeles also will not win.

The *apparent* simplicity of showing a hypothesis to be false

The initial intuition is that a hypothesis is false if a prediction derived from it is false

If the hypothesis is true, then the prediction is true
The prediction is not true
∴The hypothesis is not true

Apply this to Halley
If Halley's comet hypothesis is correct, his comet will reappear in December, 1758

Had his comet not appeared, people would have concluded that his hypothesis was wrong.

The challenge of confirmation

What seems to be the obvious way to confirm a hypothesis faces a serious problem:

If the hypothesis is true, then the prediction is true
The prediction is true
∴The hypothesis is true.

This is the form affirming the consequent, and is invalid

We can also see what is intuitively wrong with it.

> Make up a theory (a really bad one) from which you predict that sunlight feels warm.
> Check the prediction.
> Sure enough, it is true
> That doesn't make your bad theory true

The strategy for overcoming the problem of confirmation

Focus not on any prediction of a theory, but one that, ***if one did not accept the hypothesis, one would not expect to be true***

That is, one connected to the hypothesis in the following conditional:

> If the hypothesis *were not* true, then the prediction *would not* be true

Now you can invoke *modus tollens* in the confirmation:

> If the hypothesis were not true, then the prediction would not be true
> The prediction is true
> ∴The hypothesis is true