Bayes' Theorem and Normative Reasoning

"The pessimist complains about the wind; the optimist expects it to change; the realist adjusts the sails"

William Arthur Ward

Bayes' Theorem

- P(A/B) = P(B/A) x P(A) / P(B)
- Posterior probability = Likelihood x Prior Probability / Normalizing Constant
- P(B) = P(A,B) + P(notA,B)
  = P(B/A) x P(A) + P(B/notA) x P(notA)
- P(A/B) = P(B/A) x P(A) / [P(B/A) x P(A) + P(B/notA) x P(notA)]

Diagnosis Problem

A clinical test, designed to diagnose a specific illness, comes out positive for a certain patient

We are told that

1. The test is 79 percent reliable (that is, it misses 21 percent of actual cases) *
2. On average, this illness affects 1 percent of the population in the same age group as the patient
3. The test has a false positive rate of 10 percent. *

Taking this into account and assuming you know nothing about the patient's symptoms or signs, what is the probability that this patient actually has the illness?

Bayes' Theorem

$$P(A/B) = \frac{P(B/A) \times P(A)}{P(B/A) \times P(A) + P(B/notA) \times P(notA)}$$

p of illness given pos. test = (p of pos. test when illness is present * Prevalence of illness) / (p of pos test when illness is present * Prevalence of illness + p of pos test when illness is not present * 1 - Prevalence of illness)

Bayes' Theorem

$$P(A/B) = \frac{P(B/A) \times P(A)}{P(B/A) \times P(A) + P(B/notA) \times P(notA)}$$

P(disease/result)

When the test is perfectly diagnostic (i.e., is always positive when disease is there and never positive when disease isn't there)

$$P(A/B) = \frac{1 \times P(A)}{1 \times P(A) + 0 \times P(notA)} = 1$$

P(disease/result)

Prevalence of disease doesn't matter

Bayes' Theorem

P(disease/result)

$$P(A/B) = \frac{P(B/A) \times P(A)}{P(B/A) \times P(A) + P(B/notA) \times P(notA)}$$

In our uncertain world...
(i.e., when tests are not perfectly diagnostic)

P(disease/result)

$$P(A/B) = \frac{.79 \times .01}{.79 \times .01 + .1 \times .99} = .0739$$

depends on the prevalence of the illness!

Alternative Calculation

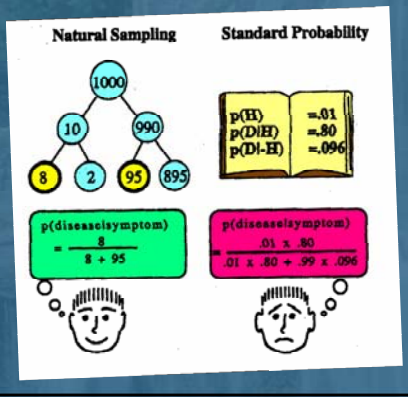
	Has illness	No illness	Totals
Test positive	790	9,900	10,690
Test negative	210	89,100	89,310
Totals	1,000	99,000	100,000

790/1690 = .0739

Why do frequencies elicit normative reasoning?

- Imagine a natural healer in a primitive society (i.e. no probability theorem). In her lifetime she has seen 1000 people, 10 of whom had the disease. Of those 10, 8 showed the symptom; of the 990 not afflicted, 95 did. A new patient has the symptoms, does she have the disease?
- *P*(disease/test result) = 8/(8 + 95) = .0777
- Gigerenzer & Hoffrage argue that the calculations are simpler (fewer steps) when presented in a frequency format.
  - fewer pieces of information need be stored – just the absolute frequencies of (disease/test) and (disease/negative test).
  - Base rates are not needed

Why do frequencies elicit normative reasoning?



Sin 1: Overconfidence

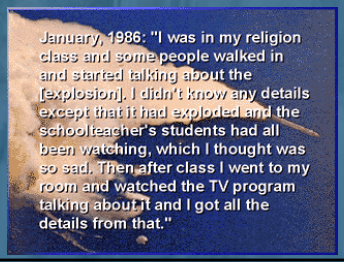
- Which city has more inhabitants?
  - Hyderabad
  - Islamabad
- How confident are you that your answer is correct?
  - 50%, 60%, 70%, 80%, 90%, 100%
- If you answer 50%, then you are guessing. If you answer 100%, then you are absolutely sure of your answer.
- People's Estimates — Actual Accuracy
  100% — 80%
  90% — 75%
  80% — 65%
- However, people are pretty accurate in judging what percentage of a set of questions they got right/wrong.

Memory and Confidence

- The power of flashbulb memories—but are they accurate?
- "For many years I have remembered how I heard the news of the Japanese attack on Pearl Harbor, which occurred on the day before my thirteenth birthday. I recall sitting in the living room of our house—we only lived in that house for one year, but I remember it will—listening to a baseball game on the radio. The game was interrupted by an announcement of the attack, and I rushed upstairs to tell my mother.
- This memory has been so clear for so long that I never confronted its inherent absurdity until last year: no one broadcasts baseball games in December!" (Neisser, 1982, p. 45).

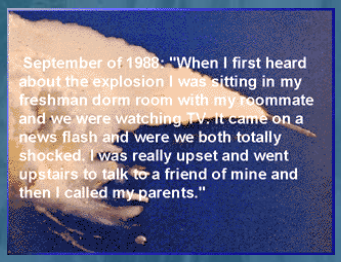
Studying Flashbulb Memories

- Where were you and what were you doing when you learned of the Challenger Crash?
- Student's report right after the crash



Studying Flashbulb Memories

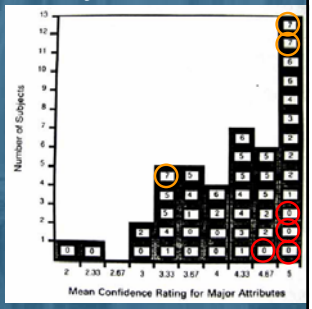
- Same student 9 months later:



- High confidence in their memories: 4.17 on a 5 point scale
- But accuracy only 2.95 on a 7 point scale

Flashbulb Memory Errors

- 25% wrong on everything
- 50% were wrong on 2/3 of what they recalled
- 7% had perfect scores
- Correlation between accuracy and confidence was not statistically significant



I was certain, but I was wrong

By Jennifer Thompson

In 1984 I was a 22-year-old college student with a grade point average of 4.0, and I really wanted to do something with my life. One night someone broke into my apartment, put a knife to my throat and raped me.

During my ordeal, some of my determination took an urgent new direction. I studied every single detail on the rapist's face. I looked at his hairline; I looked for scars, for tattoos, for anything that would help me identify him. When and if I survived the attack, I was going to make sure that he was put in prison and he was going to rot.

When I went to the police department later that day, I worked on a composite sketch to the very best of my ability. I looked through hundreds of noses and eyes and eyebrows and hairlines and nostrils and lips. Several days later, looking at a series of police photos, I identified my attacker. I knew this was the man. I was completely confident. I was sure.

I was certain, but I was wrong

I picked the same man in a lineup. Again, I was sure. I knew it. I had picked the right guy, and he was going to go to jail. If there was the possibility of a death sentence, I wanted him to die. I wanted to flip the switch.

When the case went to trial in 1986, I stood up on the stand, put my hand on the Bible and swore to tell the truth. Based on my testimony, Ronald Junior Cotton was sentenced to prison for life. It was the happiest day of my life because I could begin to put it all behind me.

In 1987, the case was retried because an appellate court had overturned Ronald Cotton's conviction. During a pretrial hearing, I learned that another man had supposedly claimed to be my attacker and was bragging about it in the same prison wing where Ronald Cotton was being held. This man, Bobby Poole, was brought into court, and I was asked, "Ms. Thompson, have you ever seen this man?"

I was certain, but I was wrong

I answered: "I have never seen him in my life. I have no idea who he is."

Ronald Cotton was sentenced again to two life sentences. Ronald Cotton was never going to see light; he was never going to get out; he was never going to hurt another woman; he was never going to rape another woman.

In 1995, 11 years after I had first identified Ronald Cotton, I was asked to provide a blood sample so that DNA tests could be run on evidence from the rape. I agreed because I knew that Ronald Cotton had raped me and DNA was only going to confirm that. The test would allow me to move on once and for all.

I will never forget the day I learned about the DNA results. I was standing in my kitchen when the detective and the district attorney visited. They were good and decent people who were trying to do their jobs -- as I had done mine, as anyone would try to do the right thing. They told me: "Ronald Cotton didn't rape you. It was Bobby Poole."

I was certain, but I was wrong

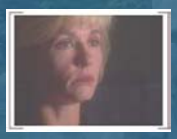
The man I was so sure I had never seen in my life was the man who was inches from my throat, who raped me, who hurt me, who took my spirit away, who robbed me of my soul. And the man I had identified so emphatically on so many occasions was absolutely innocent.

Ronald Cotton was released from prison after serving 11 years. Bobby Poole pleaded guilty to raping me.

Ronald Cotton and I are the same age, so I knew what he had missed during those 11 years. My life had gone on. I had gotten married. I had graduated from college. I worked. I was a parent. Ronald Cotton hadn't gotten to do any of that.

Mr. Cotton and I have now crossed the boundaries of both the terrible way we came together and our racial difference (he is black and I am white) and have become friends. Although he is now moving on with his own life, I live with constant anguish that my profound mistake cost him so dearly. I cannot begin to imagine what would have happened had my mistaken identification occurred in a capital case. . . .

Jennifer Thompson and Ron Cotton



The man on the left, **Ron Cotton**, who spent 11 years in prison for the rape of Jennifer Thompson. The man on the right is the rapist Bobby Poole

Examples of Overconfidence

- "A severe depression like that of 1920-1921 is outside the range of probability" *Harvard Econ. Society Weekly Letter* , Nov. 16, 1929
- "With over 50 foreign cars already on sale here, the Japanese auto industry isn't likely to carve out a big slice of the U.S. market for itself" *Business Week*, August 2, 1968
- "There is no reason anyone would want a computer in their home" Ken Olson, DEC founder, 1977
- "Forget it, Louis, no Civil War picture ever made a nickel." --Irving Thalberg's warning to Louis B. Mayer regarding Gone With the Wind
- "We don't like their sound. Groups of guitars are on the way out." --Decca Recording Company executive, turning down the Beatles, 1962

Sin 2: Magical Thinking

- Seeing a coorelation when there is none
- Selectively remembering the evidence that supports it
- "When someone is convinced of a positive correlaton, however illusory that correlation can objectively be shown to be, that person will always find new confirmations and justify why it is so" (Piattelli-Palmarini, p. 122)

Names

Jane Wyman
Gladys Knight
David Wharton
Doris Lessing
Danny Holiday
Joe Steiger
Margaret Mead
Jacob Davis
John Spitz
Peggy Fleming
Bob McCauley
Bella Abzug

An Exercise in Scientific Thinking

- An unusual phenomenon has been discovered and the researcher(s) who figure out what is going on (e.g., what is the **rule**) is a shoe-in the Nobel Prize
- One example of the phenomenon has already been discovered: **2, 4, 6**
- You can test other **examples** and Mother Nature will tell you whether the example fits the rule
- If you think you know the rule, you can **publish** it
- Mother nature cannot tell you whether you got it right or wrong—only the other students can, by voting to award you the Nobel Prize

Sin 3. Hindsight

- "A projection of new knowledge into the past accompanied by a denial that the outcome information has influenced judgment" (Hawkins & Hastie, 1990).
- Prior to Richard Nixon's trips to China and Russia in 1972, Fischhoff and Beyth asked students to consider 15 possible outcomes such as:
  - Nixon will meet Mao Tse-tung at least once
  - Nixon will see Soviet demonstrators
- The students were asked to assign a probability to each possible outcome.
- **After the trip, the students were asked, in hindsight, to remember their original (pre-trip) probabilities**
- **When the interval between the tests was two weeks, 67% thought their original estimates were closer to the truth than they really were.**
- **When a 4-8 month interval had elapsed, 84% showed hindsight bias.**

Hindsight

- A 1983 radiology study obtained chest radiographs, every four months, from 4618 patients at high risk for lung cancer.
- During the course of this six-year study, three radiologists found 92 lung tumors in the study group. In the 92 cases of identified tumors, 75 (82%) had initially been overlooked and mistakenly classified as normal.
- Subsequently, the radiologists found they could return to radiographs previously classified as normal and locate all 75 tumors.
- In visually identifying these previously missed tumors knew what type of tumor they were looking for and where to look for it.
- Without this outcome knowledge, however, those 75 tumors were not obvious.

Hindsight

- We are good at advancing explanations
- Tell me a result, and I will figure out why it happened
- Now it seems obvious

Sin 4. Anchoring

- **Stage 1: Person starts with initial idea of answer ("anchor")**
  - **Ball park estimate.**
  - **Anchor may be suggested by memory, or by something in environment.**
- **Stage 2; Person adjusts away from initial anchor to arrive at final judgement.**
- **Problem: Adjustments are generally inadequate. Final estimate is too closely tied to anchor**

Anchoring Example

- What are the last three digits of your home phone number?
- Add 400.
- Do you think Attila the Hun was defeated in Europe before or after that year (A.D.)?
- Now, in what year would you guess Attila the Hun was defeated?
- Correct answer: A.D. 451

Anchoring Example

Range of initial anchor (last 3 digits of phone number plus 400)	Average estimate of year of Attila's defeat
400-599	629
600-799	680
800-999	789
1000-1199	885
1200-1399	988

Anchoring

Kahneman & Tversky 1974

- Task: Suppose you randomly pick the name of one of the countries in the UN. What is the probability that this country will be an African country?

1. A wheel-of-fortune is spun and yields a "random number" between 1 and 100.
2. Ask whether the actual percentage of African countries in UN is higher or lower than that number (Sets anchor)

Anchoring

3. Subject is asked for their estimate of the number

- Results
  - When Stage 1 number was 65, mean estimate was 45%
  - When Stage 1 number was 10, mean estimate was 25%
- Subjects are inappropriately swayed by random anchor.

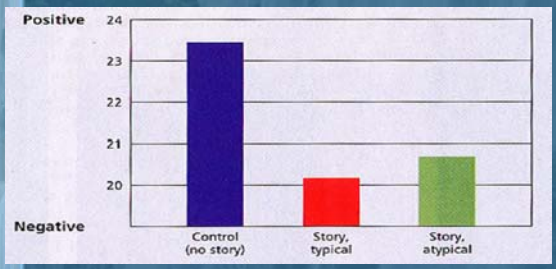
Everyday example of Anchoring and Counter-Anchoring

- Car dealer attempts to anchor you to "windshield price" on car
- Combat by anchoring on price dealership paid
- Problem with using anchoring and adjustment heuristic is sticking too close to bad anchor.

Social Anchoring

- Hamill, Wilson, and Nisbett (1980) had participants read a story about an unlikable welfare recipient. She was described as **irresponsible** and her life as bleak.
  - Some participants were told this woman was typical of welfare recipients.
  - Others were told she was atypical of welfare recipients.
- Then participants rated their attitudes about all welfare recipients.
- Even those who were told the person was atypical of welfare recipients anchored their attitudes to their attitude towards this woman.

Social Anchoring



Sin 5. Ease of Representation

- **Which of the following causes more deaths in the USA each year?**
  - **Stomach cancer**
  - **Motor vehicle accidents**
- **Most respondents select motor vehicle accidents, but stomach cancer causes twice as many deaths.**
- **The "availability" of media stories about motor vehicle deaths biases our perception of the frequency of events.**

Ease of Representation

- Which causes more deaths:
  - Accidents or strokes?
  - Homicides or diabetes?
  - All cancers combined or heart disease?
- The correct answers are on the right, but most people select the ones on the left

Ease of Representation

- Subjects asked to estimate frequency of various causes of death.

Cause	S. estimate	Truth
Tornado	564	90
Fireworks	160	6
Asthma	506	1886
Drowning	1684	7380

(rates per 200m US residents per year)

Slovic, Fischhoff & Lichtenstein (1976)

Sin 6 Probability Blindness

- "Any probabilistic intuition by anyone not specifically tutored in probability calculus has a greater than 50 percent chance of being wrong."
- Which super-lotto ticket would you pick:
  7, 12, 18, 24, 33, 45
  or
  1, 2, 3, 4, 5, 6?
- Which one is more likely to hit?
- Why do most people prefer the first ticket?

Sin 7. Reconsideration under suitable scripts

- "... a plausible and well-told story can lead us to hold as **objectively** probable events that, just minutes before, we would have considered totally improbable" (p.135)